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Dilaton tadpoles and D-brane interactions in compact spaces

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Abstract: We analyse some physical consequences when supersymmetry is broken by a set of D-branes and/or orientifold planes in type II string theories. Generically, there are global dilaton tadpoles at the disk level when the transverse space is compact. By taking the toy model of a set of electric charges in a compact space, we discuss two different effects appearing when global tadpoles are not cancelled. On the compact directions a constant term appears that allows to solve the equations of motion. On the non-compact directions Poincaré invariance is broken. We analyse some examples where the Poincaré invariance is broken along the time direction (cosmological models). After that, we discuss how to obtain a finite interaction among D-branes and orientifold planes in the compact space at the supergravity level.

Keywords: Superstrings and Heterotic Strings, D-branes, Superstring Vacua.

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1. Introduction

In recent years, some models with supersymmetry broken by a set of D-brane and orientifold planes in type II string theories have been considered [1, 2]. When these objects are located on a compact manifold, one should take into account that the Ramond-Ramond tadpoles are cancelled [3]. On the contrary the NS-NS tadpoles can remain uncacelled.[1] The presence of these tadpoles has a series of physical consequences: redefinition of the background in such a way that Poincaré invariance is broken [5], effective potentials for some moduli [6], divergences in higher string amplitudes, etc. Some supergravity solutions in the presence of these global tadpoles can be found in [7]–[9].

[1]There is the possibility of breaking supersymmetry and avoid the disk tadpoles as in [4].

In a pair of papers, Fischler and Susskind [5] showed how to deal with these NS-NS tadpoles and the divergences associated with them. In the first paper, they showed how the divergences can be eliminated by introducing a dilaton condensate. This condensate acts like a background that is a solution to the equations of motion with the inclusion of the tadpole terms. In the second paper, it was shown how to deal with small divergent handles (in our case they will be disks) and how they can be cancelled by shifting to a suitable background.[2]

In this paper we intend to re-visit some of the physics that arises from a system with global charges. As a guiding illustrative example, we take the toy model of a system of electric charges located at some points in a compact manifold but expanded in some additional non-compact directions. We will see that due to the presence of these global charges, a tadpole term is generated, having two important physical consequences. On the one hand, in order to find a solution to the electrostatic potential, Poincaré invariance should be broken. The term which breaks Poincaré invariance is proportional to the sum of the charges divided by the volume of the compact manifold. When the global charge vanishes or when we take the decompactification limit, that term goes to zero, allowing Poincaré invariant solutions.

The other effect concerns the compact manifold. The same term that breaks Poincaré invariance appears like a uniform neutralising background. This is similar to the jellium model in condense matter physics. There, the ions in a solid are replaced by a rigid uniform background of positive charge while valence electrons neutralise this background.[3] In our case, the jellium term allows to find solutions to the Poisson equation in a compact space when the total charge is not vanishing. This term changes the usual behaviour of the propagator in the compact space. The shape of this propagator is, at short distance with respect to the compactification volume, similar the the propagator in non-compact space. But at large distances, the corrections from the jellium term become important. From this redefined propagator we can derive the interactions in the compact space. As expected, the result coincide with other methods to define it, like a suitable regularization of the potential created by an infinite periodic array of images or by computing the energy of the system.

From the electrostatic analogy we pass to the D-brane case. Now the equations we would like to solve are the Einstein equations with the dilaton and Ramond-Ramond fields turned on, with the D-branes being the sources for these fields. These equations are considerably more complicated (highly non-linear) than in the simplified electrostatic case but, as we will see, the physical behaviour is similar. By integrating the dilaton equation on the compact space, we get that if the transverse space to the D-branes and/or O-planes is compact and the sum of the 'dilatonic charges' does not vanish, there is no solution for the dilaton field which preserves Poincaré invariance on the brane directions. If we suppose that the metric has a bi-warped form, i.e.: that the dependence on the non-compact coordinates in the compact space comes by a global factor (and similarly in the compact ones), one

[2]See, for more detailed explanation, [10, 11].

[3]For an introduction to the jellium model approximation see, for instance, [12].

can deduce the jellium term and a dependence between the warped factors and the dilaton field. Similar conditions have been obtained from the Einstein equations under the name of Brane Sum Rules [15]. In general, one can obtain an infinite number of these consistency conditions by taking linear combinations of them (as explained in appendix A).

Non-vanishing tadpole solutions in supergravity have been analysed previously in [7, 8]. We revisit the cosmological solutions of Dudas and Mourad [7] for the Sugimoto model [1] and construct some others by taking T-duality transformations. One of the main features of these solutions is the presence of a space-like singularity. At larger times the string coupling goes to zero and the dilaton term becomes irrelevant (the disk is a higher order term than the sphere).

From the electrostatic analogy we can also understand the D-brane interaction in compact space. The naive cylinder diagram is divergent due to the tadpole term. The divergence is coming from the sum over windings in the open string picture, i.e.: the numbers of the images in the covering space grows faster than the decay with the distance, similarly to what is happening in the Olbers paradox. One should correctly define the propagator in the compact space. A jellium type of term appears in the definition, which allows to find solutions to the propagator in the compact space. The presence of the jellium term in the momentum space has the interpretation of the absence of a propagating zero mode, making the amplitude finite and coinciding with the one defined by the sum over images when it is correctly regularised.

2. The effect of tadpoles in electrostatics

To understand the meaning of the different tadpoles due to the D-branes we will develop a toy model: electrostatics in compact spaces without boundary. As we will see, although it is a simplified model, it captures most of the physical results we want to point out for the case of D-brane systems.

The basic equation we need to solve is the Poisson equation with sources localised at some points on the manifold $\mathcal{M}$:

$$\Delta_y \phi = \sum q_i \delta(y - y_i)\,. \tag{2.1}$$

Obviously, this equation has solutions only if $\sum q_i = 0$. We will call this condition the tadpole cancellation condition. This condition comes from the integration of the above equation on the compact manifold. Our case is in some way analogous: the tensions of all the branes do not add to zero. However in that case these charged objects are extended in some non-compact extra dimensions. As we will see shortly, is this fact which allows to find solutions to the Poisson equation, even if there is a global charge in the compact space.

Since we present the electrostatic analogy for illustrative proposes, we consider that the metric on the transverse and parallel dimensions do not mix. Then the laplacian on the whole space can be split into a compact and parallel space dependence: $\Delta = \Delta_c + \Delta_p$. Let us also consider that the potential ϕ can be decomposed into a sum of a compact dependent part and a parallel dependent part, $\phi = \phi_c + \phi_p$. The Poisson equation can then be written as

$$\Delta\phi = \Delta_c\phi_c + \Delta_p\phi_p = \sum q_i\delta(y - y_i)\,. \tag{2.2}$$

By integrating this equation on the compact space, one finds that

$$V_{\mathcal{M}}\Delta_p\phi_p = \sum q_i\,, \tag{2.3}$$

where $V_{\mathcal{M}}$ is the volume of the compact manifold. The equation (2.3) reflects the existence of the tadpole. In the dimensional reduced theory it appears as a term in the effective action:

$$S = \int \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{\sum q_i}{V_{\mathcal{M}}}\phi\right). \tag{2.4}$$

It signals that the background should be re-defined and that there is no solution in the parallel dimensions which satisfies Poincaré invariance.

Let us turn back to the compact space. By using the equation (2.3) into equation (2.2), one finds that the equation to solve is not the Poisson equation in the compact space, which has no solution, but the modified one:

$$\Delta_c\phi_c = \sum q_i\delta(y - y_i) - \frac{\sum q_i}{V_{\mathcal{M}}}\,. \tag{2.5}$$

So, in the compact space, the tadpole induces a term in the Poisson equation that allows to find solutions. It can be interpreted as a constant neutralising background. We will call this term a jellium term, borrowing the name from solid state physics.

The interaction between the charges can be obtained by considering the propagator in the compact space. This propagator satisfies, as we will see in section 5, the equation

$$\Delta_c G(y) = \delta(y) - \frac{1}{V_{\mathcal{M}}}\,, \tag{2.6}$$

where in addition to the usual delta function, one has to introduce a jellium-type of term, $-1/V_{\mathcal{M}}$, which allows the equation (2.6) to have solution.

As we know, the interaction between the sources can be obtained from the propagator,

$$\mathcal{A} = \sum_{ij} q_i G(x_i, x_j) q_j\,. \tag{2.7}$$

But there is another way of computing the interaction between the charges that gives the same result. If we just try, as in the non-compact case, to consider the potential created by the other branes and to introduce a probe, the equation without jellium term has no solution. However, what one could do is to consider the solution of eq. (2.1) for a system of charges that satisfy the tadpoles and obtain the energy of the configuration:

$$E = \int_{\mathcal{M}} \nabla\phi^2 = -\sum_i q_i\phi(y_i)\,. \tag{2.8}$$

The second expression had been obtained by using the Poisson equation and the Stokes theorem. The energy will depend on the positions of the charges.

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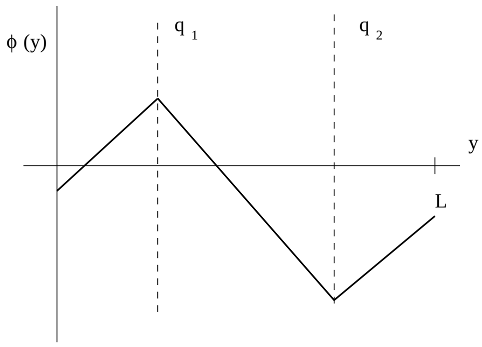

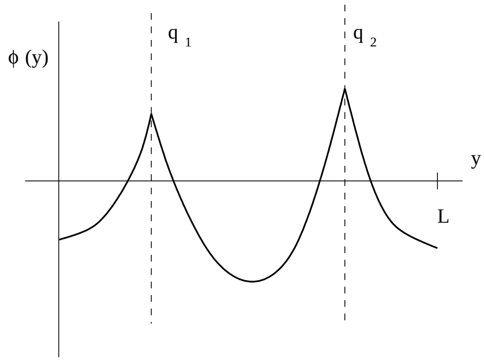

Figure 1: On the left hand side we have represented the electrostatic potential of two opposite charges on a circle. As tadpole conditions are satisfied one gets linear dependence as expected. On the right side the tadpoles are not cancelled and one finds a quadratic dependence on the distance proportional to the tadpole.

In the case where the tadpoles do not vanish, one can, by solving the equation (2.5), obtain an electrostatic potential that integrated gives the energy of the system:

$$E = \int_{\mathcal{M}} \nabla\phi^2 = -\sum_i q_i \phi(y_i) + \frac{\sum q_i}{V_{\mathcal{M}}} \int_{\mathcal{M}} \phi \,. \tag{2.9}$$

On the other hand, the electrostatic potential is given by the sum of the propagator multiplied by the charges,

$$\phi_c = \sum_i q_i G(y, x_i) \,. \tag{2.10}$$

It is easy to see that the energy of the system reproduces the correct interaction behaviour of the charges. Just substituting (2.10) into (2.9) one obtains:

$$E = -\sum_{ij} q_i G(x_i, x_j) q_j + \frac{(\sum q_i)^2}{V_{\mathcal{M}}} \int_{\mathcal{M}} G(y) \,. \tag{2.11}$$

It means that the energy of the system reproduces the interaction plus a constant that depends on the global tadpoles. Notice that if we take the decompactification limit the tadpole terms vanish. So it suggests that there is a continuous transformation from the compact to the non-compact cases, where the tadpole is no present (flux can escape to infinity).

2.1 An example: charges in a circle

Let us consider a system of two charges, q_1 and q_2, at the points y_1 and y_2 in a circle of length L. Tadpoles are satisfied if the sum of these charges is zero.

If, naively, one tries to find by the method of the images what is the potential created by these charges, one finds

$$V(y) = q_1 \sum_{n\in\mathbb{Z}} |y - y_1 + nL| + q_2 \sum_{n\in\mathbb{Z}} |y - y_2 + nL| \,. \tag{2.12}$$

which diverges unless $q_1 + q_2 = 0$. On the other hand, by solving

$$\Delta_c \phi_c = q_1 \delta(y - y_1) + q_2 \delta(y - y_2) - \frac{q_1 + q_2}{L} \tag{2.13}$$

and imposing periodicity and continuity, one finds:

- $0 \le y \le y_1$: $\phi_c(y) = \frac{y}{L}\left(q_1 y_1 + q_2 y_2 - \frac{L}{2}(q_1 + q_2)\right) - \frac{(q_1 + q_2)y^2}{2L} + q_1 y_1 + q_2 y_2 + \phi_0\,,$
- $y_1 \le y \le y_2$: $\phi_c(y) = \frac{y}{L}\left(q_1 y_1 + q_2 y_2 + \frac{L}{2}(q_1 - q_2)\right) - \frac{(q_1 + q_2)y^2}{2L} + q_2 y_2 + \phi_0\,,$
- $y_2 \le y \le L$: $\phi_c(y) = \frac{y}{L}\left(q_1 y_1 + q_2 y_2 + \frac{L}{2}(q_1 + q_2)\right) - \frac{(q_1 + q_2)y^2}{2L} + \phi_0\,.$

The potential is represented in figure 1. In the case where the tadpoles are cancelled there is no quadratic term in the potential and the electrostatic potential is described by straight lines. In the points where there is a charge the first derivative jumps. It also happens in the case where the tadpoles are not cancelled. The difference is that there are parabolic segments instead of lines, as seen in figure 1.

The propagator can also be obtained by soving the equation:

$$\Delta_c G(y) = \delta(y) - \frac{1}{L}\,, \tag{2.14}$$

where we have put the source at the origin. The solution to the above equation can be easily obtained:

$$G(y) = \frac{|y|}{2} - \frac{y^2}{2L} + C\,, \qquad |y| \le L\,. \tag{2.15}$$

Notice that for large volume the propagator reproduces the expected behaviour for non compact space, i.e.: the linear behaviour in our one dimensional case. At large distances, comparable to the compactification size, the propagator is modified to be periodic. See figure 2.

The energy of the system can be computed by using (2.9):

$$E(y_1, y_2) = -q_1 q_2 \left(|y_1 - y_2| - \frac{(y_1 - y_2)^2}{L}\right) + (q_1 + q_2)L\left(C + \frac{L}{12}\right). \tag{2.16}$$

Notice that there is a linear dependence as expected from a laplacian type behaviour. Because the space is compact there is a quadratic dependence (also in the absence of tadpoles). The tadpoles do not affect the interaction but only give a vacuum energy.

One might wonder if the divergence appearing by the introduction of the infinite array of images can be conveniently regularized and then substracted, in such a way that the resulting finite potential reproduces the physical potential and propagator obtained by the previous methods. The non-compact propagator for an infinite periodic array of charged images is

$$G_{\text{div}}(y) = \sum_{n\in\mathbb{Z}} |y + nL| = |y| + \sum_{n\in\mathbb{N}} |y + nL| + \sum_{n\in\mathbb{N}} |y - nL|\,. \tag{2.17}$$

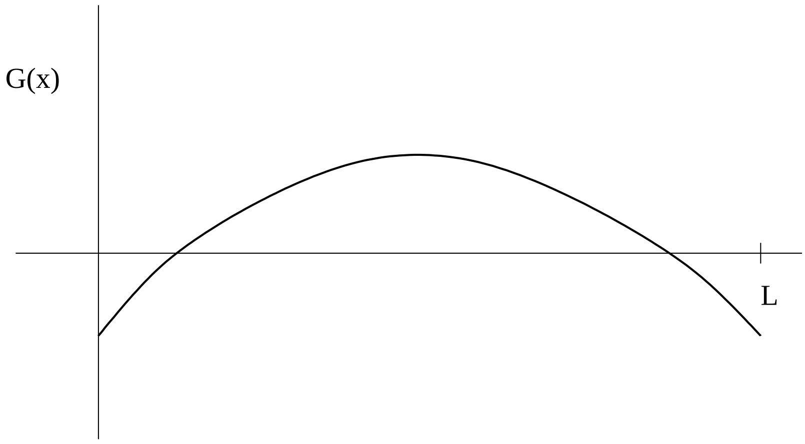

Figure 2: Propagator on a circle. At short distances the behaviour is like in non-compact space. At large distance should be modified to take into account that our space is compact.

By using (regulating and discarding the divergent part [10])

$$\sum_{n>0}(n-a) = \frac{1}{24} - \frac{1}{8}(2a-1)^2\,, \tag{2.18}$$

one obtains

$$G_{\rm reg}(y) = |y| - \frac{y^2}{L} - \frac{L}{12}\,, \tag{2.19}$$

which reproduces the interaction obtained by solving the equation (2.14) with $C = -L/12$.

It is curious to observe that if we express the propagator (2.19) in Fourier modes, we get

$$G_{\rm reg}(y) = -\frac{L}{4\pi^2}\sum_{n\neq 0}\frac{e^{2\pi i n(y/L)}}{n^2}\,, \tag{2.20}$$

i.e.: the zero mode has been completelly substracted. Therefore, its inclusion is what made the propagator (2.17) divergent.

Summarising: there are two different effects of tadpoles in compact manifolds. From the point of the reduced effective action on the non-compact space, they modify the action, meaning that the background solutions are redefined. From the point of view of the compact dimensions the tadpole appears as a necessary extra term in the equations of motion on the compact space.

3. Interpretation of NS-NS tadpoles in superstring theories

Once we have understood the electrostatic case, let us take a system of branes, antibranes and orientifold planes located at different points in a compact space. We will pay attention to the dilaton equation and the consistency conditions derived from there. The idea is to integrate the equation in the compact space and, like in the electrostatic case, obtain some consistency conditions. The physical consequences are analogous to the electrostatic case: dilaton tadpoles break Poincaré invariance along the directions parallel to the D-branes. The jellium term has also an analogy in these systems.

Similar consistency conditions can be obtained by considering the Einstein equations. These ideas have been applied in brane world scenarios under the name of brane sum rules [15]. Even more, one can consider linear combinations of these consistency conditions as explained in the appendix A and B.

3.1 Dilaton equation

In the Einstein frame, the equation for the dilaton coupled to some sources (D-branes, orientifold planes,...) has the form:

$$\partial_M \left(\sqrt{-G} G^{MN} \partial_N \phi\right) + \\ + \frac{3-p}{4}\left(e^{((p-3)/4)\phi}\sqrt{-\hat{g}}\sum_i q_i \delta^{9-p}(y-y_i) - e^{((3-p)/2)\phi}\sqrt{-g}\frac{|F_{p+2}|^2}{(p+2)!}\right) = 0\,,$$

where G_{MN} is the metric in ten dimensions, $\hat{g}_{\mu\nu}$ the induced metric on the p-brane and $q_i \sim g_s l_s^{7-p}$ are its tensions. Finally, y_i are the positions where the different objects are located.

Let us consider the case of a ten dimensional manifold with a topology $R^{p+1} \times \mathcal{M}$ where $\mathcal{M}$ is a compact manifold of dimension $9-p$. We consider Dp-branes expanding the non-compact coordinates and located at some points in the compact part. Let us take the metric

$$G_{MN} = g^{nc}_{\mu\nu}(x,y) dx^\mu dx^\nu + g^c_{ij}(x,y) dy^i dy^j\,. \tag{3.1}$$

With this ansatz for the metric and integrating the equation for the dilaton we obtain the following consistency condition:

$$\int_{\mathcal{M}} \partial_\mu \left(\sqrt{-G} G^{\mu\nu}\partial_\nu \phi\right) = \frac{p-3}{4}\sum_i q_i \sqrt{-g^{nc}(x,y_i)}\, e^{((p-3)/4)\phi(x,y_i)} + \\ + \frac{3-p}{4}\int_{\mathcal{M}} e^{((3-p)/2)\phi}\sqrt{-g}\frac{|F_{p+2}|^2}{(p+2)!}\,. \tag{3.2}$$

An immediate physical consequence is that if the right hand side of the equation is non-vanishing, one has to require that $\partial_\mu \phi \neq 0$, i.e.: the dilaton field has to break the Poincaré invariance along the directions of the branes.

3.2 (Bi)warped metric

Let us take a less general metric with warped factors in both the non-compact and compact coordinates,

$$G_{MN}(x,y) = e^{\Omega(y)} g_{\mu\nu}(x) + e^{B(x)} h_{ij}(y)\,. \tag{3.3}$$

Notice that this case includes the usual warped cases, like the supersymmetric solutions, where $B(x) = 0$ and $g_{\mu\nu}(x) = \eta_{\mu\nu}$.

We can consider, like in the electrostatic case, that the dilaton can be decomposed in a compact and a non-compact dependence: $\phi(x,y) = \varphi(x) + \phi(y)$. The action for the dilaton in the Einstein frame is of the form:

$$S = -\frac{1}{4\kappa^2}\int \sqrt{-G}\left(e^{-\Omega}|\partial\varphi(x)|_g^2 + e^{-B}|\partial\phi(y)|_h^2\right). \tag{3.4}$$

With this ansatz the dilaton equation can be decomposed into a set of different equations:

1. There is a linear relation between the warped factor on the compact coordinates and the $\varphi(x)$ of the form:

$$2(7-p)\partial_\mu B(x) = (p-3)\partial_\mu \varphi(x)\,. \tag{3.5}$$

   This relation comes from the factorisation of the x and y dependence of the dilaton equation.

2. A similar equation can be obtained from the previous one and the Einstein equations for mixed indices (see appendix B). This equation relates the warped factor on the non-compact coordinates and the $\phi(y)$. Assuming $\partial_\mu B \neq 0$ [4], we have

$$2(3-p)\partial_i \Omega(y) = (p-7)\partial_i \phi(y)\,. \tag{3.6}$$

3. By integrating on the compact manifold, we can obtain the dilaton equation in the non-compact coordinates:

$$V_{\mathcal{M}}\partial_\mu(e^{B(x)} g^{\mu\nu}\partial_\nu \varphi(x)) = \frac{(p-3)}{4}(T-E)\,, \tag{3.7}$$

   where T and $V_{\mathcal{M}}$ are the analogues of the tadpole term and volume term in the electrostatic case:

$$T = \sum_i q_i e^{-8/(p-7)\Omega(y_i)} \tag{3.8}$$

   and

$$V_{\mathcal{M}} = \int_{\mathcal{M}} \sqrt{g_c} e^{((p-1)/2)\Omega(y)}\,. \tag{3.9}$$

   The E term is due to the coupling of the RR fields to the dilaton in the Einstein frame:

$$E = \int_{\mathcal{M}} \sqrt{h} e^{-((p-3)/2)\phi(y)-((p+1)/2)\Omega(y)} |F_{p+2}|^2 \tag{3.10}$$

   We will discuss bellow the physical interpretation of these equations.

4. The dilaton equation in the compact manifold is

$$\begin{aligned}\partial_i(\sqrt{h} e^{((p+1)/2)\Omega} h^{ij}\partial_j \phi(y)) &= \frac{p-3}{4}\Bigg(e^{((p-3)/4)\phi+((p+1)/2)\Omega}\sum_i q_i \delta(y-y_i) - \\ &\qquad - e^{((3-p)/2)\phi-((p+1)/2)\Omega}\sqrt{h}|F_{p+2}|^2\Bigg) - \\ &\quad - \frac{p-3}{4} e^{((p-1)/2)\Omega}\sqrt{h}\frac{T-E}{V_{\mathcal{M}}}\,.\end{aligned} \tag{3.11}$$

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[4] Below there is a discussion for $p=3$, where $\partial_\mu B = 0$.

3.3 RR field equation

We are working up to the disk level in the string coupling. At this order, the branes do not interact among them, since the cylinder contribution is at higher order. In this static approximation, we can take the RR $p+1$ form potential, in an appropriate gauge, to be

$$C_{p+1} = C(y)dx^0 \cdots dx^p \,. \tag{3.12}$$

Then, the equation of motion for the RR field is

$$\partial_i \left(e^{-((p+1)/2)\Omega + ((3-p)/2)(\phi+\varphi)} \sqrt{h} h^{ij} \partial_j C \right) = \left(e^{((p-7)/2)B(x)} \sqrt{-g(x)} \right) \sum_n q_n^{RR} \delta^{9-p}(y-y_n) \,, \tag{3.13}$$

where q_n^{RR} are the RR charges for the D-brane and orientifold planes.

3.4 Physical implications

Now we extract some physical implications from the above equations:

- The volume V of the compact space has a warped factor dependence $\Omega(y)$. That can be seen directly by reducing the action on the compact space with the above ansatz. That dependence also appears in the supersymmetric case when the NS-NS and RR charges are not cancelled locally.

- Also the tadpole T has a warped factor dependence. This dependence comes from the volume and dilaton dependence of the coupling. As in the illustrative example in section 2, if the volume goes to infinity or the tadpole vanishes a solution to the equations of motion can be found where Poincaré symmetry is preserved. When tadpoles are cancelled like in T-dual models to type I one can construct the solution by the harmonic function method by taking into account all the images. The tadpole T in function of this harmonic function is of the form:

$$T = \sum_i q_i H^{-1}(y_i) \,. \tag{3.14}$$

  One can check that T is equal to zero in these case because the harmonic function has pole at the points where the D-branes are located, for $p < 7$.

- For a system of Dp-branes with $3 < p < 7$ the above equations tell us that:
  - From equation (3.5), the cosmological evolution of the compact coordinates is related to $\partial_\mu \varphi(x)$ with the same sign. This means that if the compact coordinates are getting smaller, the string coupling becomes weaker.
  - As in the supersymmetric case, the warped factor $\Omega(y)$ is proportional to the dilaton dependence on the compact space (up to a constant shift) due to the equation (3.6).

- The case $p = 3$ is special. From equation (3.5) we get $\partial_\mu B(x) = 0$. Then, for this case, the equation (3.7) tell us that one can choose a constant value of the dilaton. That was expected, since D3-branes and anti D3-branes do not couple to the dilaton.

- For $p < 3$, the x-derivatives in the equation (3.5) have opposite sign. For a cosmological solution with the compact dimensions becoming larger in time the string coupling decreases.

- Contrary to the case of the NS-NS tadpole, the RR tadpole has to cancel, as can be seen by integrating the equation (3.13) on the compact manifold to obtain $\sum_n q_n^{RR} = 0$.

- Since the left hand side of the equation (3.13) is independent of the brane coordinates x, we have that, at the disk level in perturbation theory, the expansion of the universe on the brane given by the metric $g_{\mu\nu}$ has to be conveniently compensated by the expansion of the moduli $B(x)$, such that

$$e^{-(\frac{p-7}{2})B(x)}\sqrt{-g(x)} = \text{const}\,. \tag{3.15}$$

- A similar analysis can be carried out by taking the Einstein equations. This conditions are known as Brane Sum Rules [15]. One can consider linear combinations of these condition to obtain a continuous set of consistency conditions as explained in the appendix A.

4. Examples

In this section we will consider some examples where dilaton tadpoles are present. In particular, we will be interested in finding the solutions to the supergravity equations with the disk terms present. First we review the more familiar supersymmetric case (T-dual configurations to type I string) where the supersymmetric solution only depends on an harmonic function which can be expressed by the method of the images. In this case summing over all the images gives a finite result, since there are no tadpoles. Then we will consider the case where NS-NS tadpoles are not cancelled but there are no transverse directions to the brane, such that the RR tadpole is cancelled locally. The only physical consequence will be the lack of Poincaré invariant solutions on the brane. In particular we will re-visit the cosmological solution of the Sugimoto string [1] found by [7]. Then we will construct some models with lower dimensional branes by taking T-dualities of the above solution.

4.1 Supersymmetric case

Let us consider as an illustration a T-dual model to type I theory on a torus. The Dp-branes are located at some points of a T^{9-p} torus. There are also 2^{9-p} orientifold planes with RR charge and tension equal to 2^{p-5} in Dp-brane units. In order to cancel the RR-charge 16 Dp-branes are needed. One can cancel the tension and RR charges locally if $p \geq 5$ by putting 2^{p-5} Dp-branes on each Op-plane.

The solution to the supergravity equations with these sources in a compact space can be constructed from the harmonic function in the compact space:[5]

$$H(y) = 1 + \sum_i q_i \sum_{\vec{m}_i \in \mathbb{Z}^{9-p}} |\vec{y}_i + \vec{m}_i L|^{p-7} \,. \tag{4.1}$$

Notice that this function does not suffer from the tadpole divergence due to the fact that the sum of all the charges is zero (so the constant term vanishes). The metric, dilaton and RR field are determined by as the harmonic funcion (4.1). There is a problem close to the orientifold planes because the H change its sign, and the metric is not defined with a negative harmonic functions. That is expected to be cured by non-perturbative effects in the same way as for the $O6$-plane non-perturbative effects change the Taub-NUT metric with negative charge into the Atiyah-Hitchin one. These effects are expected to be important close to the orientifold planes but the solution constructed from the above harmonic function is expected to be correct far away from the negative tension objects.

Finally, notice that this solution respects Poincaré symmetry along the directions parallel to the branes as we expected from tadpole cancellation conditions.

4.2 Sugimoto model

The first example we consider is the Sugimoto model [1]. The idea is, like the orientifold construction of type I string, to start from type IIB and introduce an orientifold plane with opposite charges (NS-NS and R-R) with respect the type I orientifold. In order to cancel the R-R tadpoles one introduces 16 dynamical antiD9-branes (32 if the orientifold images are taken into account). These antibranes break down the supersymmetries that were preserved by the orientifold, such that no supersymmetry is left unbroken. From the open string projection one can see that the orientifold keeps the symmetric representation for the massless gauge bosons and the antisymmetric one for the gauginos. All together gives a USp(32) gauge group with fermions transforming in the antisymmetric representation.

As NS-NS charges are not cancelled, there is a tadpole term in the effective action. This configuration is the extreme case where the compact space is just a point, and all the non-trivial behaviour due to the NS-NS tadpoles is reflected in a breaking of the Poincaré invariance at the disk level. The re-definition of the background due to these terms affects the closed string propagator beyond one loop. But at the level of the disk, there are already effects of the tadpole term in the ten dimensional background. They were analysed by Dudas and Mourad [7]. The effective potential is of the form (in Einstein frame):

$$\int \Lambda e^{-3\phi/2} \,, \tag{4.2}$$

where $\Lambda = 32T_9$. This potential looks very similar like a quintessence potential, but the solution has a very different behaviour. We will consider homogeneous and isotropic spatially flat solutions. The solution has a space-like singularity and the dilaton potential energy and the ten dimensional scalar curvature go to zero at the infinite future. The

[5] We take an squared torus to symplify notation.

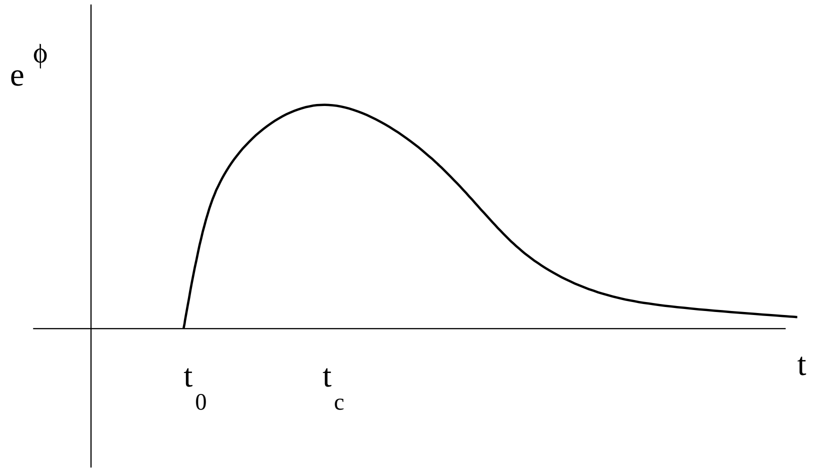

Figure 3: Dilaton behaviour.

universe at long times suffers a deceleration. It is easy to demonstrate that the solution is unique up to two parameters that determine the time location of the (big bang) singularity and the normalization of the string coupling.

The solution in the Einstein frame has the following form [7]:

$$\begin{aligned} e^{\phi} &= e^{\phi_0}(\sqrt{\Lambda}t - c_0)^{2/3} e^{((-3(\sqrt{\Lambda}t - c_0)^2)/4)} \\ ds_E^2 &= -(\sqrt{\Lambda}t - c_0)^{-1} e^{-3\phi_0/2} e^{((9(\sqrt{\Lambda}t - c_0)^2)/8)} dt^2 + (\sqrt{\Lambda}t - c_0)^{1/9} e^{(((\sqrt{\Lambda}t - c_0)^2)/8)} dx_{||}^2 , \end{aligned} \tag{4.3}$$

where c_0 and ϕ_0 are the constant parameters of the solution. The value of the c_0 is related to the position of the singularity by $t_0 = -c_0/\sqrt{\Lambda}$.

The dilaton grows from zero to a maximal value at $t_c = (2/3 - c_0)/\sqrt{\Lambda}$. The value of the dilaton at the maximum is $e^{\phi_c} = e^{\phi_0}(2/3)^{2/3} e^{-1/3}$. Then it starts to decrease to zero, as seen in figure 3.

The scalar curvature of the solution when $t_0 = \phi_0 = 0$ is (for other values the solution is physically equivalent; see figure 4):

$$R = \frac{-81t^4 + 252t^2 - 16}{72te^{9t^2/8}} . \tag{4.4}$$

The maximum of the Ricci scalar is at t_c, the maximum also of the dilaton potential. Close to the critical point t_c, where the dilaton gets its maximum, the universe is accelerating. The energy is basically concentrated in the potential of the dilaton that acts as a cosmological constant. At the singularity, e^{ϕ} goes to zero and the scalar curvature diverges as $(t - t_0)^{-1}$. The dilaton potential goes to zero.

For larger times, the dilaton potential energy goes to zero, the universe deccelerates. Strings are very weakly coupled and the system is driven to free strings. The solution is not flat in that case because there is some energy in the dilaton kinetic term.

The Poincaré limit can be obtained when the tadpole Λ goes to zero. In that case the singularity goes to infinity, the metric becomes the constant flat metric $\eta_{\mu\nu}$ and the dilaton gets a constant value $e^{\phi} = e^{\phi_0}(c_0)^{2/3}$

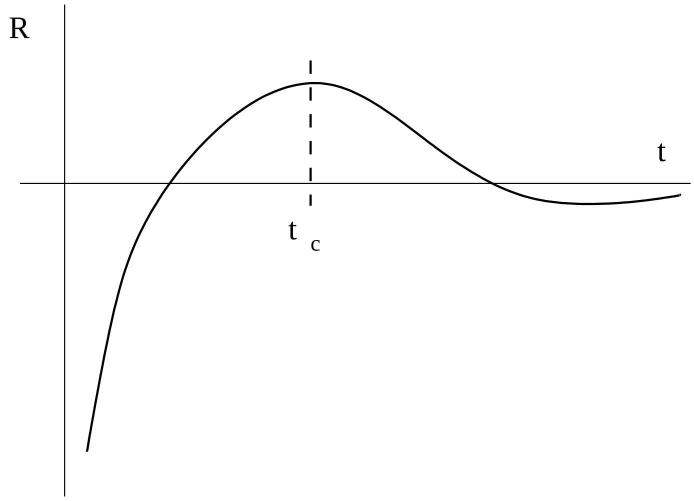

Figure 4: Ricci scalar for the Dudas-Mourad solution.

4.3 T-duality

Since the metric is translationally invariant on the spatial coordinates $x_{||}$, we can perform T-duality transformations along these directions. Consider compactifying one direction: $x^9 \simeq x^9 + 1$. The T-dual metric to (4.3) is, in the Einstein frame:

$$\begin{aligned} ds_E^2 = \; & -(\sqrt{\Lambda}t - c_0)^{-1} c_1^{-3/2} e^{((9(\sqrt{\Lambda}t - c_0)^2)/8)} dt^2 + (\sqrt{\Lambda}t - c_0)^{1/9} e^{(((\sqrt{\Lambda}t - c_0)^2)/8)} d\hat{x}_{||}^2 + \\ & + (\sqrt{\Lambda}t - c_0)^{2/3} e^{-(9/8)(\sqrt{\Lambda}t - c_0)^2} dx_9^2 \end{aligned} \tag{4.5}$$

and the dual dilaton becomes

$$e^{\tilde{\phi}} = e^{\tilde{\phi_0}} (\sqrt{\Lambda}t - c_0)^{7/9} e^{-(5/8)(\sqrt{\Lambda}t - c_0)^2} \,. \tag{4.6}$$

This solution corresponds to an smeared distribution on the 9-direction of orientifold 8-planes and anti-D8-branes. Notice that the solution has no singularities in the internal space. The solution where the D8-branes are localised on the top of each of the orientifold planes was constructed in [8], finding singularities in the internal space where also the dilaton diverges. It would be nice to see how general is the relation between localised solutions and singularities in the compact manifold.

With only one direction T-dualized, the scalar curvature and the dilaton profile are very similar to the original model. We can proceed performing further T-dualities on the extra spacial directions. In the string frame, our original metric has the expression

$$ds^2_{\text{string}} = -a^2_{\text{string}}(t) dt^2 + b^2_{\text{string}}(t) dx^2_{||} \,, \tag{4.7}$$

with

$$a^2_{\text{string}}(t) = e^{-\phi_0} (\sqrt{\Lambda}t - c_0)^{-2/3} e^{(3/4)(\sqrt{\Lambda}t - c_0)^2} \,, \tag{4.8}$$

$$b^2_{\text{string}}(t) = e^{-\phi_0/2} (\sqrt{\Lambda}t - c_0)^{4/9} e^{-(\sqrt{\Lambda}t - c_0)^2/4} \,. \tag{4.9}$$

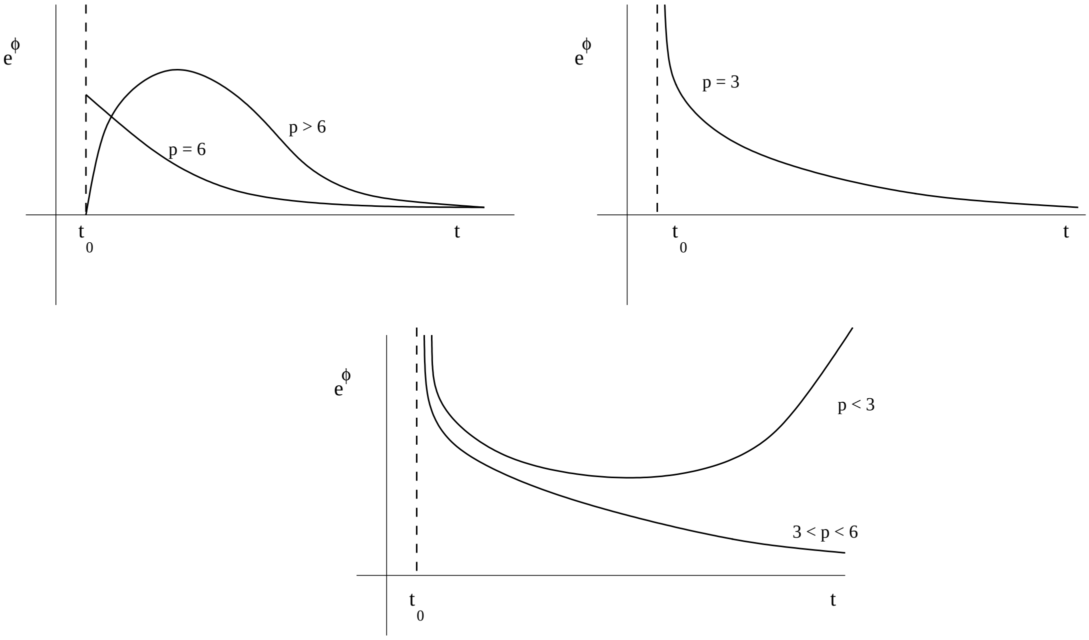

Figure 5: Dilaton behaviour for different solutions depending on the dimension of the brane.

We can perform $n = 9 - p$ T-dualities on the longitudinal directions $\vec{x}_{||}$ to obtain additional brane transverse directions $\vec{x}_{\perp}$. According to Busher's rules, the following background is also a solution of the equations of motion, corresponding to having $p = 9 - n$ D-branes and orientifold planes dislocalized in the n compactified directions $\vec{x}_{\perp}$:

$$
\begin{aligned}
ds^2_{\text{string}} &= -a^2_{\text{string}}(t)dt^2 + b^2_{\text{string}}(t)dx^2_{||} + b^{-2}_{\text{string}}(t)dx^2_{\perp}\,, && (4.10)\\
e^{\tilde{\phi}} &= e^{\phi(t)} b^{-n}_{\text{string}}(t) && (4.11)\\
&= e^{((p-5)/4)\phi_0}(\sqrt{\Lambda}t - c_0)^2)^{(2/9)(p-6)} e^{((3-p)/8)(\sqrt{\Lambda}t - c_0)^2}\,. && (4.12)
\end{aligned}
$$

Notice that for $3 < p < 6$, the string coupling diverges at $t = t_0$ and decays exponentially for $t > t_0$. At $p = 3$, the coupling follows only a power law dependence in t. This case corresponds to a system of dislocalized orientifold 3-planes and anti-D3 branes along the six transverse directions. In this way, RR flux is locally cancelled and the transverse translational invariance is preserved. Since in the Einstein frame the dilaton does not couple to 3-branes, its decreasing is much slower than the scalar curvature's, which in the Einstein frame goes as

$$
R(t) = \frac{(2 - 9(\sqrt{\Lambda}t - c_0)^2)}{(\sqrt{\Lambda}t - c_0)^{5/3} e^{(3/4)(\sqrt{\Lambda}t - c_0)^2}}\,. \qquad (4.13)
$$

When $p > 6$ we have the same qualitative behaviour as in the Sugimoto solution in ten dimensions: at large times and close to the singularity the solution goes to weak coupling. The case $p = 6$ is interesting as the solution near the singulaty has a non-vanishing coupling but at large distances keeps the exponential decay. For $p < 3$ the dilaton has a divergent behaviour at the singularity and at the infinite future.

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Another interesting point is that the solutions we describe here do not satisfied the relation (3.5). That is because for deriving that relation one assumes that the sources are localised, or at least have some dependence, on the compact manifold. However for these T-dual solutions that is no longer true, and one can find solutions where there is no dependence on the compact coordinates. For localised or semi-localised solutions, like the ones in [8], the relations hold.

5. D-brane interactions

5.1 Introduction

Let us consider some D-branes expanding some non-compact space coordinates and being localised at some points on a compact manifold. Ramond-Ramond tadpoles impose strong constraints on the allowed configurations: without orientifold planes, the number of branes should be equal to the number of antibranes. As brane-antibrane forces are attractive and the the brane-antibrane pair can anihilate each other, one expects the supersymmetric vacuum to be the final configuration. This is not the case in the presence of orientifold planes (for example, for T-dual configurations of the type I superstring), where a net RR charge for the D-branes is necessary to cancel the negative RR charge of the orientifold planes.

If the configuration is supersymmetric, the D-brane interactions are absent; this is easy to check by the tree level exchange of closed strings. Things can be slightly different when the disk terms are present, since they are the dominant terms in perturbation theory. A very clear example is the case of a brane and antibrane, at just opposite points on a compact space. The situation is unstable at one loop due to the attractive force between the two branes. However the disk terms make the compact space to expand, taking both branes far appart, such that non-supersymmetric sectors become very massive and decouple.

Let us for the moment forget about the disk terms and pay attention to the one loop amplitudes. From the open string point of view supersymmetry allows a cancellation between bosons and fermions. That means that the field which represents the distance between the branes has a flat direction. But when supersymmetry is not present, there is a one loop potential for the scalar fields, which represent the distances between the two branes. From the closed string point of view it means an interaction between the D-branes at tree level.

Let us first review the interaction between a Dp-brane and and anti-Dp-brane in flat space when the transverse directions are non-compact. By going to the one loop open string channel that interaction is just the vacuum energy of the system [14, 10],

$$V(y_i,\theta) = -V_{p+1}(i)^p \int_0^\infty \frac{dt}{t}(8\pi^2\alpha'\, t)^{-((p+1)/2)} e^{-(((y^2)/2\pi\alpha')-\pi)t}\eta^{-12}(it)\theta_{11}^4\left(\frac{it}{2}, it\right), \quad (5.1)$$

where y is the modulus of the vev of the scalar fields living on the brane, parametrizing the transverse directions. In the closed string interpretation y is the distance between the two branes. At long distances with respect the string length, the amplitude is better reproduced by performing a Poisson re-summation,

$$V(y_i,\theta) = -V_{p+1}(i)^p \int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-((p+1)/2)} e^{-(((y^2)/2\pi\alpha')-\pi)t} t^4 \eta^{-12}\left(\frac{i}{t}\right)\theta_{11}^4\left(\frac{1}{2},\frac{i}{t}\right). \tag{5.2}$$

The main contribution is for $t \to 0$,

$$V(y_i,\theta) = -V_{p+1}(i)^p \int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-((p+1)/2)} e^{-((y^2)/2\pi\alpha')t} 16t^4 \simeq y^{p-7}. \tag{5.3}$$

As expected from the supergravity analysis, at long distances the interactions between the branes are mediated by massless closed string fields. The potentials produced by these fields are the potentials expected from the solutions of the laplacian operator in the directions transverse to the branes. The amplitude is not divergent except at short distances, when the open string tachyon develops. This divergence can be cured by analytic continuation. We can obtain a real part (the potential) and an imaginary part associated to the tachyon that reflects that the system can decay to another system with lower energy (in this case the tachyon).

After this short review of the one-loop amplitude for non-compact transverse space, we compactify $9-p$ transverse directions to the D-branes in a torus. In the open string one loop amplitude, one should take into account the contributions from the winding modes,

$$V(y_i,\theta) = -V_{p+1}(i)^p \int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-((p+1)/2)} \sum_{m_i} e^{-t/(2\pi\alpha')\sum_i (y_i+m_iL_i)^2} \frac{\theta_{11}^4(it/2,it)}{e^{\pi t}\eta^{12}(it)}. \tag{5.4}$$

If we naively commute the integral and the sum on the windings we arrive to the following expression for the potential when the two D-branes are far apart:

$$V(y) = k\sum_{m_i}\left[\sum_i (y_i+m_iL_i)^2\right]^{p-7} \tag{5.5}$$

for some constant k. Notice that the transposition of the sum and the integral allows to interpret our result as the sum over all the images, as in electrostatics. However, as the number of compact directions is $9-p$, at a 'distance' in the transverse space r, there are r^{8-p} images, which grows faster with the distance that the decreasing of the interaction.[6] So we have found an IR divergence due to the massless closed string modes. In the open string picture this divergence is an UV effect.[7]

This divergence was expected, since the system, although is free of RR tadpoles, suffers from NS-NS tadpoles. These tadpoles appear at the disk level and indicate that the background should be redefined [5]. But as we have shown in the example of electrostatics, there is another effect for a tadpole in a compact space: the zero mode for a massless field has to be excluded from propagating. It is the inclusion of the zero mode which is causing

[6] That is similar to the Olbers paradox, where the number of light sources at a given distance is of the same order as the luminosity, so the total luminosity remains constant.

[7] See the appendices of [13]

the divergence in equation (5.5). In the next subsection, we will derive the correct equation for the propagator of massless fields in compact spaces, such that the exclusion of the zero mode is guaranteed.

5.2 Propagator in compact spaces

Here we will review how to obtain the propagator using path integral methods and will verify the volume dependent extra term for its differential equation.

Let us consider a D-dimensional compact manifold $\mathcal{M}$ provided with the euclidean metric g. It has the finite volume $V = \int dx^D \sqrt{g}$ and no boundaries. Consider the euclidean action of a massless field ϕ coupled to the source J:

$$S(\phi, J) = \int dx^D \sqrt{g} \left(\frac{1}{2} |\partial\phi|^2 - \phi J \right) . \tag{5.6}$$

In order to compute the generating functional

$$Z[J] = \int \mathcal{D}\phi \; e^{-S(\phi,J)} , \tag{5.7}$$

we define its path integral measure by introducing a basis of ortonormal eigen-functions $\{\phi_n\}_{n\in\mathbb{Z}}$, satisfying

$$\int dx^D \sqrt{g} \phi_n \phi_m = \delta_{n,m} \tag{5.8}$$

and

$$\nabla^2 \phi_n = \frac{1}{\sqrt{g}} \partial_i (\sqrt{g}\, g^{ij} \partial_j \phi_n) = -\omega_n^2 \phi_n \qquad n \in \mathbb{Z} . \tag{5.9}$$

Notice that $\omega_n \geq 0$. If $\omega_0 = 0$, then we have a zero mode. Normalisation determines that $\phi_0 = \frac{1}{\sqrt{V}}$. Observe that in the infinity volume limit the zero mode vanishes. The existence of this zero mode is due to the symmetry $\phi \to \phi + c$ for the operator ∇^2. In a more general situation, zero-modes always come associated to symmetries of the action. If a mass term is introduced into the action (5.6), the new operator in the quadratic action is $\tilde{\nabla}^2 = \nabla^2 - m^2$ and its eigen-values simply get shifted by $\tilde{\omega}^2 = \omega^2 + m^2$.

Using the fact that for any function $\phi(y)$ defined on $\mathcal{M}$ we have that

$$\phi(y) = \sum_n a_n \phi_n(y) , \tag{5.10}$$

we can define the path integral measure by $\mathcal{D}\phi \equiv \prod_n \sqrt{2\pi} da_n$. The path integral can now be evaluated:

$$\begin{aligned} Z[J] &= \int \Pi_n (\sqrt{2\pi} da_n) \exp \sum_n \left(-\frac{1}{2} \omega_n^2 a_n^2 + a_n J_n \right) \\ &= \left(\int da_0 e^{a_0 J_0} \right) \prod_{n \neq 0} \left(\int_{-\infty}^{\infty} \sqrt{2\pi} da_n e^{-(1/2)\omega_n^2 a_n^2 + ((J_n^2)/(2\omega_n^2))} \right) \\ &= \delta(J_0) (\det{}' - \nabla^2)^{-1/2} \exp \left(-\frac{1}{2} \int d^D x \; d^D y J(x) G(x-y) J(y) \right) , \end{aligned} \tag{5.11}$$

where we have shifted the integration variable $a_n \to a_n + \omega_n^{-2} J_n$ and introduced the Green function

$$G(x-y) = -\sum_{n\neq 0} \frac{\phi_n(x)\phi_n(y)}{\omega_n^2}\,, \tag{5.12}$$

which satisfies

$$\nabla^2 G(x) = \delta^D(x) - \frac{1}{V}\,. \tag{5.13}$$

One can check that shifting $\phi(x) \to \phi(x) - \int d^D y\ G(x-y)J(y)$ into (5.6), only for the Green function satisfying (5.13) the generating functional (5.11) is reproduced.

For localised charges, the source does not have a constant component and $J_0 = 0$. In this case, the quadratic action (5.6) does not depend on the zero-mode a_0, The situation is similar to a gauge symmetry, where there is an integral on a local field which does not appear in the action. As in the local symmetry situation, we have to "gauge-fix" the translational symmetry responsible of the zero-mode. Following the usual techniques, we add the following factor in the path integral:

$$\frac{1}{V}\int d^n x \int dc\ \delta\left(\phi(x) - c\right) = 1\,. \tag{5.14}$$

Permuting some integrals, such that the one on the translational symmetry, $\int dc$, is left at the end, we have

$$\begin{aligned} Z[J] &= \int dc \int da_0 \frac{1}{V} \int d^D x\ \delta\left(\frac{a_0}{\sqrt{V}} - c + \cdots\right) \cdot \\ &\quad \cdot \prod_{n\neq 0}\left(\int_{-\infty}^{\infty} \sqrt{2\pi} da_n e^{-(1/2)\omega_n^2 a_n^2 + ((J_n^2)/(2\omega_n^2))}\right) \\ &= \left(\int dc\right)\sqrt{\frac{V}{\det'(-\nabla^2)}}\exp\left(-\frac{1}{2}\int d^n x\ d^n y J(x) G(x-y) J(y)\right)\,. \end{aligned} \tag{5.15}$$

5.3 Physical consequences in D-brane systems

We are going to discuss about the physical consequences of the zero mode in the interaction amplitude between D-branes located at different points in a compact manifold $\mathcal{M}$. We know that there are two ways of understanding this amplitude, as a one loop amplitude of open strings or as a tree level amplitude of closed strings. When the sources for the closed strings are at distances greater that the string length, the amplitude then has the structure:

$$\mathcal{A} = \sum_{ij} q_i G(x_i, x_j) q_j\,, \tag{5.16}$$

where q_i are the (NS-NS and RR) charges for the D-branes and $G(x,y)$ is the propagator in compact space of the massless modes mediating the interaction at large distances.

From the previous discussion we have seen that the propagator in compact space has a volume dependence. That has several consequences depending on how we look at this:

- The correct Green equation for a massless propagator in a compact space is equation (5.13). The volume dependent term is necessary in order to find a solution to the equation (5.13).

- We can see that the volume dependent term has the effect of subtracting the zero mode, since

$$\delta^D(x-y) - \frac{1}{V} = \sum_{n\neq 0} \phi_n(x)\phi_n(y)\,. \tag{5.17}$$

Also, form equation (5.12) we can see that the zero mode has been explicitly subtracted from the sum. Its inclusion would induce a divergence on the amplitude.

- In toroidal compactifications, if we write the propagator with theta functions, we can see that the absence of the zero mode is equivalent to adding a constant counter-term, of the form:

$$G \sim \int_0^\infty dx \left[\Pi_{i=1}^D \theta_3\left(\frac{y_i}{L_i}, \frac{ix}{L_i^2}\right) - 1\right]. \tag{5.18}$$

It is straightforward to check out that the -1 in the previous integral is cancelling the contribution of the zero mode. If we introduce a regulator mass m for the massless closed string state, one can see that the counter-term in the interaction amplitude between the two D-branes is proportional to

$$\mathcal{A}_{ij} \sim \lim_{m\to 0} \frac{q_i q_j}{mV}\,. \tag{5.19}$$

Notice that in the non-compact case the amplitude becomes finite and there is no need to introduce the counter-term.

- For the total amplitude, summing over all the D-brane contributions, the counter-term is of the form:

$$\mathcal{A} = \sum_{ij} \mathcal{A}_{ij} \sim \lim_{m\to 0} \frac{(\sum_i q_i)^2}{mV}\,. \tag{5.20}$$

Firstly notice that in the non-compact limit the counter-term vanish and the amplitude is finite. Notice also that the amplitude is finite if the sum over all the charges is zero, i.e.: tadpole is cancelled. in this case, the open string amplitude gives the correct behaviour.

6. Conclusions

In this paper we have anayzed two different effects by the NS-NS tadpoles. On the one hand, we have seen that the tadpole generically induces solutions which are not Poincaré invariant on the brane longitudinal non-compact dimensions. We analyzed cosmological solutions of the Sugimoto model and its T-duals. As expected, there is an space-like singularity at some time t_0. Later on, the string coupling goes to zero and the space-time becomes flat, except for $p < 3$, where the exponential dilaton grows with time. Since this solutions where obtained applying T-duality, the p-branes and orientifold planes are dislocalized on the compact space. It would be interesting to analyse solutions where the D-branes and Orientifolds are located at particular points in the compact space, as it is done in [8].

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Our approach is perturbative in the string coupling. We have seen that, already at the disk level, which is the lowest order where NS-NS tadpoles appear, there are non-trivial relations among the metric on the brane, the warp factor in front of the metric of the compact space, and the dilaton dependence on the brane coordinates. These are the equations (3.5) and (3.15) respectively. We should keep in mind that these relations are derived at the disk level and for the bi-warped metric ansatz (3.3).

The same kind of effect happens for the dependence on the compact coordinates. The NS-NS tadpole produces an extra term in the equations of motion of the massless fields, a "jellium" type of term. For the bi-warped metric ansatz (3.3), the extra term is the last one on the right hand side of equation (3.11) Then, we expect the tadpole to modify also the background field profile on the compact space.

The second important consequence of the tadpole that we wanted to stress concerns the interaction among the charged objects on the compact manifold. First we have observed that the periodic interaction potential for D-branes constructed by the method of images is generically divergent. A closer analysis shows the zero-mode as the responsable for this divergence. Using path integral methods, we have observed that the proper two-point correlator function in a compact space satisfies a "modified" Poisson equation, the equation (5.13), where the volume dependent extra term has the effect of subtracting the zero-mode contribution form the propagator, dealing with a perfectly finite and consistent interaction amplitude for D-branes in compact spaces. This new term is related to the tadpole in the background field equations of motion. When the two charged objects are very close with respect the size of the compact space, the propagator behaves as the propagator in non-compact spaces without tadpoles. But for distances comparable to the compactification scale, the effect of the tadpoles becomes relevant, as we have seen for the illustrative model of electric charges in a circle.

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A. Total derivative operators in compact spaces

Let us consider $A(y)$ a function from compact space to the real numbers. For every function $f(A)$ one can define the operator

$$\begin{aligned} \Delta(f(A)) &= \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}g^{ij}\partial_j f(A)) \\ &= f'(A)\Delta(A) + f''(A)(\partial A)^2 \,, \end{aligned} \tag{A.1}$$

where $(\partial A)^2 = g^{ij}\partial_i A\partial_j A$ and $f'(A) = df(A)/dA$. In particular we can always write the term

$$\Delta A + g(A)(\partial A)^2 = \frac{1}{f'(A)}(f'(A)\Delta A + f''(A)(\partial A)^2) \,, \tag{A.2}$$

where $g = f''/f'$. That allow as to write

$$\Delta A + g(A)(\partial A)^2 = e^{-\int g}\Delta f\,. \tag{A.3}$$

Particular cases of interest are:

i) When $g(A) = r$ is a constant,

$$\Delta A + r(\partial A)^2 = e^{-rA}\Delta(e^{rA})\,. \tag{A.4}$$

ii) When $g(A) = (n-1)/A$, then

$$\Delta A + \frac{n-1}{A}(\partial A)^2 = \frac{A^{1-n}}{n}\Delta(A^n)\,. \tag{A.5}$$

Linear combinations and consistency conditions

Now, let us consider a set of equations of the form:

$$\begin{aligned} \Delta A + r_1(\partial A)^2 &= F_1 \\ &\ddots \\ \Delta A + r_N(\partial A)^2 &= F_N\,, \end{aligned} \tag{A.6}$$

where r_i are constants and F_i are general formulae.

One can consider a general linear combination of the equations, that can be written as

$$\Delta A + \alpha(\partial A)^2 = \frac{\sum \lambda_i F_i}{\sum \lambda_i}\,, \tag{A.7}$$

where $\alpha = \sum r_i\lambda_i / \sum \lambda_i$. That can be written by using the results from the previous subsection as

$$\Delta(e^{\alpha A}) = e^{\alpha A}\frac{\sum F_i\lambda_i}{\sum \lambda_i}\,. \tag{A.8}$$

We can integrate the relations on the compact manifold without boundary and get a set of consistency conditions:

$$\int_{\mathcal{M}} e^{\alpha A}\sum_i F_i\lambda_i\,. \tag{A.9}$$

There are a RP^{n-1} set of consistency conditions (the linear combinations up to a global factor).

B. Ricci tensor for (bi)warped metrics

We take two warped factors $\Omega(y)$ and $B(x)$. With this ansatz for the metric the Ricci tensor in ten dimensions is

$$\begin{aligned} R^{10}_{\mu\nu} &= R^{p+1}_{\mu\nu} - \frac{9-p}{2}\left(\nabla_\mu\nabla_\nu B + \frac{1}{2}\partial_\mu B\partial_\nu B\right) - g_{\mu\nu}\frac{e^{\Omega-B}}{2}\left(\Delta\Omega + \frac{p+1}{2}(\partial\Omega)^2\right), \\ R^{10}_{\mu i} &= 2\partial_\mu B\partial_i\Omega\,, \\ R^{10}_{ij} &= R^{9-p}_{ij} - \frac{p+1}{2}\left(\nabla_i\nabla_j\Omega + \frac{1}{2}\partial_i\Omega\partial_j\Omega\right) - g_{ij}\frac{e^{B-\Omega}}{2}\left(\Delta B + \frac{9-p}{2}(\partial B)^2\right). \end{aligned} \tag{B.1}$$

And the Ricci scalar is

$$R^{10}=e^{-\Omega}\left[R_x-(9-p)\left(\Delta B+\frac{10-p}{4}(\partial B)^2\right)\right]+e^{-B}\left[R_y-(p+1)\left(\Delta\Omega+\frac{p+2}{4}(\partial\Omega)^2\right)\right]. \tag{B.2}$$

If we consider that the dilaton field can be decomposed in a compact and a non-compact dependence: $\phi(x,y) = \varphi(x) + \phi(y)$. The action for the dilaton in the Einstein frame is of the form:

$$S=\frac{-1}{2\kappa^2}\int\sqrt{-G}\frac{1}{2}(e^{-\Omega}(\partial\varphi(x))^2+e^{-B}(\partial\phi(y))^2)\,. \tag{B.3}$$

This term has a contribution to the energy momentum tensor:

$$\begin{aligned}
T^{10}_{\mu\nu} &= \frac{1}{2}\partial_\mu\varphi(x)\partial_\nu\varphi(x)-\frac{1}{4}e^{\Omega}g_{\mu\nu}(e^{-\Omega}(\partial\varphi(x))^2+e^{-B}(\partial\phi(y))^2)\,,\\
T^{10}_{\mu i} &= \frac{1}{2}\partial_\mu\varphi(x)\partial_i\phi(y)\,,\\
T^{10}_{ij} &= \frac{1}{2}\partial_i\phi(y)\partial_j\phi(y)-\frac{1}{4}e^{B}g_{ij}(e^{-\Omega}(\partial\varphi(x))^2+e^{-B}(\partial\phi(y))^2)\,.
\end{aligned} \tag{B.4}$$

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